Income inequality and savings: a reassessment of the relationship in cointegrated panels

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Abstract

The effect of inequality on savings has remained an open empirical issue despite of several decades of research. Results obtained in this study indicate that inequality and private consumption are both $I(1)$ nonstationary variables that are cointegrated with each other. This implies that previous empirical research may have produced spurious results on effect of inequality on savings or consumption by assuming that inequality would be a stationary variable. According to estimation results, income inequality has had a negative effect on private consumption in Central-European and Nordic countries. Results for Anglo-Saxon countries are inconclusive.
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1 Introduction

The effect of savings on capital accumulation and growth has been one of the fundamental research topics in economics starting from the very beginning of economic sciences. According to Smith (1776), an increased division of labor raises productivity, but savings govern capital accumulation, which enables production growth. In the 18th century, only rich people saved. Therefore, economic growth was possible only when there were enough rich people in society. However, according to Keynes (1964), inequality of income would slow down economic growth. Keynes argued that

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marginal consumption decreases as income of individual increases, and thus, aggregate consumption depends on changes in aggregate income. Because demand is the basis of investments, and because inequality lowers aggregate consumption, inequality of income would diminish economic growth. In neo-classical growth models, income distribution determines the level savings and thus the level of capital accumulation (Solow 1956; Kaldor 1957).

In addition to the approach of classical economics, there exists several theories describing the effect of income inequality on savings or consumption. These include the permanent income hypothesis by Friedman (1957), life-cycle hypothesis by Ando and Modigliani (1963), which was further developed to include intergenerational transfers by Kotlikoff and Summers (1981), savings under liquidity constraints by Deaton (1991), and political-economy models (e.g. Alesina and Perotti (1994)).

Although theoretical research spans through several decades, the effect of income inequality on savings remains an open empirical question. This is because empirical studies have produced controversial results on the effect of income inequality on savings. In one of the most recent panel econometric studies, Leigh and Posso (2009) estimate the effect of the income shares of the top 1% on the percent share of gross savings on GDP and find no statistically significant effect on inequality to national savings. Similarly, Schmidt-Hebbel and Servén (2000) find no statistically significant effect on inequality on percent share of savings on GDP using several different measures of income inequality. However, Smith (2001) finds that inequality, measured with Deininger and Squire's (1996) Gini index, has a statistically robust positive effect on the percent share of savings on GDP. Cook (1995) finds the same effect at work in less developed economies. Li and Zou (2004) find that the relation between inequality and savings might vary across different countries, using Deininger and Squire's (1996) Gini index and ratio of savings to GDP.

All the empirical studies summarized above have assumed that income inequality, measured either by Gini index or by share of income earned by different income classes, is a stationary variable. However, Malinen (2011) has recently obtained results according to which the data generating process of income distribution might be driven by a stochastic trend. That is, income inequality could be a $I(1)$ nonstationary variable. If this result would hold in general, it would offer an explanation for the controversy in previous empirical studies. This is because regressing a stationary variable against an $I(1)$ variable(s) can lead to spurious regressions (Stewart 2010). In empirical studies, savings is usually measured as a percent of GDP. If both savings and GDP are $I(1)$ variables and cointegrated of order 1, dividing them with each other will result to a stationary variable, namely savings as a percent of GDP or savings as a ratio of GDP. Thus, if inequality would be $I(1)$ variable and savings as a ratio of GDP a stationary $I(0)$ variable, regressing savings against inequality could give spurious results. Even if both variables would be $I(1)$ and cointegrated, many regressors are biased, like OLS, or inconsistent, like conventional GMM, when applied to panel cointegrated data (Kao and Chiang 2000). Thus, it is possible that previous aggregate studies assessing the effect on inequality to savings may have produced biased results.

This study accounts for these sources of bias with panel cointegration methods and data on the income shares of the top 1%, which is used to proxy the distribution of income. This measure is found to follow broader measure of income inequality, like the Gini index, very well (Leigh 2007). Changes in national and private savings are measured with the gross national savings and total expenditure of private consumption. According to panel unit root tests, the income share of the top 1% and gross national savings and private consumption are found to be $I(1)$ variables. Income share of the top 1% is also found to be cointegrated with private consumption using panel cointegration testing methods. Results of panel cointegration tests are inconclusive for possible cointegration between gross savings and top 1% income share. The real GDP per capita and aggregate savings like the real GDP per capita and private consumption are also found to be cointegrated. This implies that ratio of savings and private consumption to GDP would be a stationary variable and that previous research has been likely to produce spurious results on the effect of inequality on savings.

Rest of the paper is organized as follows. Section 2 gives the theoretical and empirical background for the study. Section 3 describes the data and presents the results of unit root tests. Section 4 reports the results of cointegration tests and section 5 gives the estimation results. Section 6 concludes.

2 Theoretical and empirical considerations

2.1 Baseline theories on the effect of income inequality on savings

In classical economic theory, the form of the individual saving function determines the effect of income inequality on savings. When the saving function is linear or concave, distribution of income and wealth converge toward equality (Stiglitz 1969). If the saving function is convex, the marginal propensity to save increases with income.

According to permanent income hypothesis, individuals with low-income have higher propensity to consume and that small changes in income, or its distribution, do not affect on consumption decisions of households (Friedman 1957). Life-cycle hypothesis argues that, if bequests are luxury, saving rates should be higher among wealthier individuals (Kotlikoff and Summers 1981). According to models with liquidity constraints, if borrowing constraints affect mostly poorer households, redistribution from wealthy to poor households makes borrowing constraints less likely to bind, thus lowering aggregate saving (Deaton 1991). In political-economy models, more unequal income distribution may create demand for more redistribution through taxation and redistribution, and if the saving function of individuals in the economy is convex, i.e. the rich save more, this will diminish aggregate savings through diminished incomes of the rich (Alesina and Perotti 1994).

Theories on saving behavior have usually assumed that neither the production nor the consumption processes would include stochastic parts. However, this may not have been a realistic assumption as many economic variables tend to have some stochastic process affecting them. In a pioneering work, Hall (1978) formulated a model of consumption where the behavior of households were determined by a stochastic trend. Hall argued that the conditional expectation of future marginal utility is only a function of today's level of consumption, which means that marginal utility would be a random walk. If marginal utility would be a linear function of consumption, then, naturally, consumption would also be driven by a stochastic trend. According to to Hall, $c_t = \lambda c_{t-1} + \epsilon_t$ would be a good enough approximation of stochastic behavior of consumption under the life cycle and permanent income hypotheses. Here ϵ_t would summarize the impact of all new information coming available to the consumer at period t.

The formulation of Hall (1978) implied that assuming a nonstationary consumption does not alter the implications of the life cycle - permanent income hypothesis for income inequality, i.e., that earners of high income have higher propensity to save. However, it is also realistic to assume that income itself is driven by a stochastic process, and that income might have a linear relation with consumption. This would lead to nonstationary consumption and asset processes.

2.2 Stochastic growth, savings and liquidity constraints

Stochastic growth models assume that some part of the aggregate production is driven by a stochastic trend. General approach in the literature has been to assume uncertainty that enters the aggregate production function in some form. In the first systematic analysis of stochastic growth, Brock and Mirman (1973) studied uncertainty in neoclassical framework. In discrete time, their aggregate production function was given by

$$Y(t) = F(K(t), L(t), z(t)), \tag{1}$$

where $z(t)$ is the stochastic productivity term that dtermines how productive a given combination of capital and labor is in producing the unique final good of the economy and it is usually assumed to follow a Markow chain process with values in the set $\mathcal{Z} \equiv \{z_1, ..., .z_N\}$. To generalize, the production function function can be written as $y_t \equiv f(k(t), z(t))$, where $k(t) \equiv K(t)/L(t)$. Consumption and savings decisions of households are made after observing the realization of the stochastic shock for time t, $z(t)$, which means that saving behavior is affected by stochastic components.

Deaton (1991) has examined how liquidity constraints affect national savings, when incomes are driven by random walk with drift.[1] He finds that, when income is a random walk and borrowing constraints are binding, it is undesirable for households to undertake any smoothing of consumption implying that consumption equals income. However, Deaton (1991) assumes that the interest rate is higher than the consumers discount rate. If the interest rate equals the consumers discount rate, stochastic income process and borrowing constraints lead to a result where propensity to consume is lower in higher income levels (Seater 1997; Travaglini 2008).

2.3 Empirical findings

In the pioneering work by Hall (1978), consumption was assumed to be stochastic, but not affected by income. Nonetheless, his US data gave some evidence that recent levels of income have an effect on aggregate consumption. Campbell and Deaton (1989) found that consumption and lagged income are positively correlated implying that if income is stochastic, so is consumption.

[1] Assuming a drift in income relation is intuitive and necessary, because income tends to grow over time. Thus, just variations around that trend are assumed to be stochastic.

King *et al.* (1991) tested the hypothesis of balanced growth under uncertainty, i.e., that consumption, investments, and output are cointegrated $I(1)$ nonstationary variables. Their data on post-war US supported the hypothesis. Sarantis and Stewart (1999) found that income-consumption ratios are nonstationary in OECD countries using so called traditional panel unit root tests. Their result implies that income and consumption are not cointegrated. This result was supported by Cook (2005) using a panel LM unit root test allowing for structural breaks. However, neither of these panel unit root testing procedures allowed for cross-sectional dependence. Romero-Ávila (2009) showed that when cross-sectional dependency is taken into account, OECD consumption-income ratios are stationary.

There is a lot of research on the relation between aggregate consumption and income, but studies on the relationship between income inequality and aggregate consumption have been rare or non-existent. This obviously relates to the assumption that income inequality would be a stationary variable, which would make such statistical comparisons meaningless (see introduction). But, if income inequality is found to be driven by a stochastic process, then aggregate comparisons would the only consistent way to assess the relationship between inequality and consumption or savings. Thus, we turn on testing the time series properties of the included variables.

3 Data and tests of time series properties

3.1 Data

In this study we use the the top 1% income share of population to proxy the income distribution in different countries. In recent years there has been a growing interest towards building a long time series on the evolution of top income shares of population. Beginning from the work of Piketty (2003), top income series have been developed for several developed countries. This measuring technique makes it possible to construct substantially longer time series from the evolution of the distribution of income than what would be possible using the Gini index or similar aggregate measures. Leigh (2007) has also demonstrated that the top 1% income share series have a high correlation with other measures of income inequality, like the Gini index. The dataset on top 1% income shares gathered by Leigh is a primary source of data in this study.

However, after Leigh (2007) published his dataset there have been additions to the pool countries from which a historical dataset of the evolution of top incomes is available. Roine and Waldenström (2010) have used a data collected from several different sources in their analysis of common trends and shocks in top income series. Their dataset on top incomes is extensive, but it has one caveat. They do not have continuous observations from all countries, and so they are forced to extrapolate over some observations. This is problematic, because it is likely that extrapolation of over just one observation will alter the time series properties of the observed variable. Because Roine and Waldenström do not explicitly describe the observation over which they are extrapolating, one has to look for the individual sources from where the time series is obtained to know where the missing observations lie. After observing the individuals sources of data, we include two countries from the dataset by Roine and Waldenström, which are Norway and Finland.[2]

The dataset of this study consists of observations on 13 countries, which are: Australia, Canada, Finland, France, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland, United Kingdom and United States. Nine of these countries form the baseline data on which we have observations from all included variables. These countries are: Canada, Finland, France, Netherlands, Norway, Sweden, Switzerland, United Kingdom and United States.

The endogenous variables used in this study are gross savings and private con-

[2]For these two countries, the data on top 1% income share comes from the dataset by Roine and Waldenström (2010). For Norway, we can check the continuity of the time series data from the original source. In Finland's case the original data, allegedly spanning from 1920 to 2005, could not be checked, but we could check that the data is continuous at least from 1966 onwards. Panel unit root tests are done also without Nordic countries, and because extrapolation would have the biggest impact on those tests, the bias created by possible extrapolation over some values of top 1% series in Finland is diminished.

sumption. The data on these is obtained from AMECO database held by the European Comission. AMECO is used, because it has more extensive time series coverage on the variables in question than, for example, the dataset of World Bank. AMECO does not include data on private savings, but because private consumption is the mirror image of private savings, it should not make any difference which of these variables is used. Both variables are measured in aggregate terms to minimize the effects that some third variable might have on the relation between inequality and savings. Changes, for example, in fertility may have an effect on variables that are measured in per capita terms without affecting the income share of the top 1%. Thus, expressing gross savings and private consumption in per capita terms could add stochastic elements to the time series of those variables that are unrelated to the relation that those variables may have with income inequality.

The control variables included in the estimation are the real gross domestic product per capita, the dependency rate, and the interest rate, which are likely to be the most common control variables used in the literature. The real GDP per capita is historically thought to proxy the expected lifetime wealth of residents in a country (Cook 1995). Level of income may also have an direct effect on savings. The dependency rate is used to control for possible changes in the saving patterns across the life-cycle of individuals, and it measures the ratio of population of under 15 and over 64 years of age. Interest rates may affect individuals propensity to save by making saving more profitable or *vice versa*. 1960 is the first year included in the dataset and the last year included in the dataset varies with different groups of countries. The data on dependency is obtained from World Banks World Development Indicators and the data on interest rate is from the database of the International Monetary Fund.

The data on interest rates varies across countries in the dataset. Baseline rate is the discount or the bank rate, i.e., the rate at which central banks lend to deposit money banks. However, for some countries observations for this variable are not available for the whole time period. That is why there are some differences in the indicators of interest rates between groups of countries. In Finland, Sweden, Norway, and Switzerland, the central bank rate is used. In Canada, United Kingdom, and USA, the treasury bill rate is used, which gives the rate at which short-term securities are issued or traded in the market. In France and Netherlands, the money market rate is used, which gives the rate on short term lending between financial institutions. Although using different indicators of interest rates is not optimal, all these indicators should reflect changes in the interest rate at which consumers loan from and deposit money to banks.

3.2 Unit root testing

To test for possible unit roots, four different panel unit root tests are used. The first two are the so called traditional and the last two the so called second generation panel unit root tests. The traditional panel unit root tests, by Im *et al.* (2003) and the panel version of the ADF test by Maddala and Wu (1999), are based on the following regression:

$$\triangle y_{it} = \rho_i y_{i,t-1} + \delta_i + \eta_i t + \theta_t + \epsilon_{it}, \tag{2}$$

where δ_i are individual constants, $\eta_i t$ are individual time trends, and θ_t are the common time effects. The tests rely on the assumption that $E[\epsilon_{it}\epsilon_{js}] = 0 \ \forall \ t, s$ and $i \neq j$, which is required for calculating common time effects. Thus, if the different series are correlated, the last assumption is violated.

The second generation tests by Pesaran (2007) and Phillips and Sul (2003) are based on the regression

$$\triangle y_{it} = \rho y_{i,t-1} + \eta_i t + \alpha_i + \delta_i \theta_t + \epsilon_{it}, \tag{3}$$

where α_is are the individual constants, $\eta_i t$ are the individual time trends, and θ_t is the common time effect, whose coefficients, δ_i, are assumed to be non-stochastic, measure the impact of the common time effects of series i, and ϵ_{it} is assumed to be normally distributed with mean zero and covariance of σ^2 over t. Cross-sectional dependence is allowed through the common time effects and ϵ_{it} is assumed to be independent of ϵ_{js} and θ_s for all $i \neq j$ and s, t. The null hypothesis in all tests is that $\rho_i = 0 \; \forall \; i$ and the alternative hypothesis is that the majority of the series are stationary.

Table 1 presents the results of the four panel unit root tests for top 1% income share. Tests have been applied to three different datasets. The first includes the data on three countries with the longest continuous time series in the dataset by Leigh (2007). The second dataset includes the nine countries on who we have data for all the included variables. The third dataset includes observations from seven countries excluding Nordic countries from 1983-2002.[3] This is because, according to Roine and Waldenström (2010), there is a trend break in the series of the top 1% income share in 1991 in Nordic countries. There is a trend break in the top 1% income share series in Anglo-Saxon countries, including Australia, Canada, UK and USA, in 1982, in Central European countries, including France, Switzerland, and Netherlands, in 1976, and in Asian countries, including Japan, in 1983. So, to make sure that the results of unit root tests are not driven by structural breaks, tests are run using only those countries that should not have breaks in their top 1% income share series in the tested period.

Table 1: Panel unit root tests for the series of top 1% income share

variable	IPS	ADF	Pesaran	PS
top 1%, 1925-1998	3.689	6.489	0.403	15.743
	(0.999)	(0.953)	(0.657)	(0.203)
top 1%, 1960-1996	4.831	3.148	0.235	2.812
	(0.999)	(0.998)	(0.593)	(0.999)
top 1%, 1983-2002	-0.302	16.670	-0.281	17.636
	(0.381)	(0.273)	(0.389)	(0.127)

The tested equation is: $\triangle y_{it} = \rho_i y_{i,t-1} + \delta_i + \eta_i t + \theta_t + \epsilon_{it}$. All variables are tested in logarithms. Probabilities of the test statistics appear in parentheses. In all other tests, except in the PS test, lag lengths were determined using Schwarts information criterion (SIC). Dataset of 1925-1998 includes 7 countries and 518 observations, the dataset of 1960-1996 includes 9 countries and 333 observations, and the dataset of 1983-2002 includes 7 countries and 140 observations.

[3]Countries included in the test are: Australia, France, Japan, New Zealand, Switzerland, United Kingdom, and USA.

According to all tests in all datasets, the logarithmic top 1% income share would be a $I(1)$ nonstationary process, even when possible structural breaks are accounted for. Table 2 presents the results of panel unit root tests for other variables. Because gross savings is expressed in constant Euros and private consumption in purchasing power parities, two series of real GDP per capita series are included in the dataset.

Table 2: Panel unit root tests for other included variables, 1960-1996

variable	IPS	ADF	Pesaran	PS
gross savings	2.207	2.144	-0.186	4.79
	(0.986)	(0.999)	(0.426)	(1.000)
consumption	12.532	0.0645	0.221	7.53
	(1.000)	(1.000)	(0.587)	(0.960)
GDP, ppp	11.006	14.039	0.347	5.331
	(1.000)	(1.000)	(0.636)	(0.994)
GDP, eur	3.912	19.453	2.420	11.210
	(1.000)	(0.364)	(0.992)	(0.796)
dependency	-5.077	82.382	1.005	50.92
	(<.001)	(<.001)	(0.843)	(<.001)
interest rate	0.717	13.937	-1.144	19.97
	(0.764)	(0.731)	(0.126)	(0.220)

The tested equation is: $\triangle y_{it} = \rho_i y_{i,t-1} + \delta_i + \eta_i t + \theta_t + \epsilon_{it}$. All variables are tested in logarithms. Probabilities of the test statistics appear in parentheses. In all other tests, except in the PS test, lag lengths were determined using Schwarts information criterion (SIC). Testing includes 9 countries and 333 observations.

According to all tests, series of gross savings, private consumption, interest rates, and both versions of real GDP per capita would be $I(1)$ processes. 3 out of 4 tests reject the null hypothesis of nonstationarity of the dependency rate, which implies that it would be a trend-stationary variable.

4 Cointegration tests

4.1 Testing with the whole data

The methods for testing for cointegration in panel data have developed very rapidly during the first decade of the 21st century. One of the most commonly used cointegration testing method has been the residual based panel cointegration test by Pedroni (2004). The limitation of Pedroni's test is that it assumes independence of cross-sections, an assumption, which is likely to be violated in econometric cross-country studies.[4] Cross-sectional correlation, if not accounted for, may bias the results towards rejecting the null of no cointegration, whereas structural breaks can bias the results towards acceptance of the null. To account for both of these biases, a panel cointegration test developed by Banerjee and Carrion-i-Silvestre (2006) is used. It controls for cross-sectional dependency by common factors, and for breaks by allowing for shifts in the level, slope, trend, and cointegration vector. Possible endogeneity of regressors is controlled for in the same way as in panel DSUR, i.e., by including leads and lags of differenced explanatory variables in estimation (see the appendix). A more thorough explanation of the test by Banerjee and Carrion-i-Silvestre (2006) is presented in the appendix.

Table 3 presents the results of the panel cointegration test by Banerjee and Carrion-i-Silvestre (2006) between the top 1% income share and gross savings, and the top 1% income share and private consumption. The test allows for level and cointegration vector shifs.[5]

According to the results of the $Z_{\hat{t}_{NT}}(\hat{\lambda})$ test presented in table 3, gross savings and the income share of top 1% as well as private consumption and top 1% income share would be cointegrated at the 5% level, even when possible structural breaks are taken into account. According to the $Z_{\hat{\rho}_{NT}}(\hat{\lambda})$ test, only gross savings and the top 1% income share would be cointegrated. However, Banerjee and Carrion-i-Silvestre (2006) note that the $Z_{\hat{t}_{NT}}(\hat{\lambda})$ statistic should be preferred over the $Z_{\hat{\rho}_{NT}}(\hat{\lambda})$ statistic, because the former has considerably better size and power properties especially in small samples. Thus, we rely more on the results of the $Z_{\hat{t}_{NT}}(\hat{\lambda})$, and conclude that both gross savings and private consumption seem to be cointegrated with the top 1% income share.

Table 4 presents the results of the panel cointegration test by Banerjee and Carrion-

[4]There are, for example, only few countries that avoided the downturn of 2008 that started from the U.S.
[5]Estimation done with Gauss. We are grateful to Carrion-i-Silvestre for providing the program code.

Table 3: Banerjee & Carrion-i-Silvestre's cointegration test for gross savings, final consumption and the income share of top 1%

	gross savings	private consumption
Pedroni model with a trend		
$Z_{\hat{t}_{NT}}(\hat{\lambda})$	-3.952	-2.255
	(<.0001)	(0.0121)
$Z_{\hat{\rho}_{NT}}(\hat{\lambda})$	-3.084	-0.548
	(0.0010)	(0.292)
Level and slope trend shifts		
$Z_{\hat{t}_{NT}}(\hat{\lambda})$	-5.849	-1.860
	(<.0001)	(0.0314)
$Z_{\hat{\rho}_{NT}}(\hat{\lambda})$	0.403	2.564
	(0.6565)	(0.9948)
Trend and cointegrating vector shifts		
$Z_{\hat{t}_{NT}}(\hat{\lambda})$	-5.706	-1.678
	(<.0001)	(0.047)
$Z_{\hat{\rho}_{NT}}(\hat{\lambda})$	1.527	3.917
	(0.937)	(0.999)
countries	9	9
years	1960-1996	1960-1996
observations	333	333

Model with level shift includes time trend and a level and slope trend shift. Model with a cointegrating vector shift includes time trend and cointegration vector shifts.

Table 4: Banerjee & Carrion-i-Silvestre's cointegration test for gross savings, final consumption and GDP per capita

	gross savings	private consumption
Pedroni model with a trend		
$Z_{\hat{t}_{NT}}(\hat{\lambda})$	-5.683	-5.494
	(<.0001)	(<.0001)
$Z_{\hat{\rho}_{NT}}(\hat{\lambda})$	-3.090	-6.399
	(0.0010)	(<.0001)
Level and slope trend shifts		
$Z_{\hat{t}_{NT}}(\hat{\lambda})$	-5.736	-4.042
	(<.0001)	(<.0001)
$Z_{\hat{\rho}_{NT}}(\hat{\lambda})$	-0.244	0.8242
	(0.4038)	(0.7951)
Trend and cointegrating vector shifts		
$Z_{\hat{t}_{NT}}(\hat{\lambda})$	-5.125	-4.898
	(<.0001)	(<.0001)
$Z_{\hat{\rho}_{NT}}(\hat{\lambda})$	-0.0155	-0.3939
	(0.4945)	(0.3469)
countries	9	9
years	1960-1996	1960-1996
observations	333	333

Explanatory variable in both tests is GDP per capita. Model with level shift includes time trend and a level and slope trend shift. Model with a cointegrating vector shift includes time trend and cointegration vector shifts.

i-Silvestre (2006) between the real GDP per capita and gross savings, and between the real GDP per capita and private consumption.[6] The test allows for level and cointegration vector shifs.[7]

According to the results of table 4, the GDP per capita and gross savings would be cointegrated of order 1. Results also imply cointegration of the GDP per capita and private consumption.

[6]GDP per capita in Euros is used in the for gross savings as the gross savings series is also presented in Euros. GDP per capita in ppp is used in the test for private savings for the same reason.

[7]Estimation done with Gauss. We are grateful to Carrion-i-Silvestre for providing the program code.

4.2 Testing for the cointegration rank

As presented in section 3.1, it is likely that several variables have an effect on the relationship between inequality and savings and/or consumption. One of the obvious drawbacks of the residual tests, like the one presented above, is that they cannot identify the number of cointegrating vectors between the variables. If we have just two variables, this is not a problem, because, then there can be only 1 cointegrating vector. With three variables, there can be two cointegrating vectors, with four variables three cointegrating vectors, etc. The rank of cointegration affects estimators, because some estimators, like panel DSUR, are based on the single equation approach meaning a single cointegration relationship is assumed. If there are two or more cointegration relationships between the variables, the asymptotic properties of the estimators derived under the assumption of one cointegration relation are no longer valid. In addition, estimators allowing for multiple cointegrating vectors usually assume that the cointegration rank is homogenous across the countries included in the panel, like the estimator of Breitung (2005) used in this study.

For these reasons we use the panel trace cointegration test developed by Larsson and Lyhagen (2007) to test for the cointegration rank of models involving several explanatory variables. Their test is based on the likelihood ratio test developed by Johansen (1995). The general model on which Larsson ja Lyhagen's test is based can be written as

$$\triangle Y_{it} = \mu_i + \Pi Y_{i,t-1} + \Sigma_{k=1}^{m-1} \Delta Y_{i,t-k} + \epsilon_{it}, \tag{4}$$

where $\Pi = \alpha_{ik}\beta_{kj}$, Π and Γ_k are of order $Np \times Np$ and μ_i and ϵ_{it} are of order $Np \times 1$, and $\epsilon_t = (\epsilon'_{1t}, \epsilon'_{2t}, ..., \epsilon'_{nt})$ is assumed to be multivariate normally distributed with mean zero and covariance matrix Ω.

It is assumed that matrix Π has a reduced rank of Nr, $0 \leq r \leq p$, and can be decomposed as $\Pi = \alpha_{ik}\beta'_{kj}$ (Larsson and Lyhagen 2007). α_{ik} is assumed to be unrestricted, but

$\beta_{kj} = 0 \; \forall i \neq j$. The unrestricted α means that different panel units can be dependent, but because of the restriction on β, these dependency relations can only appear in the short run. In other words, cointegrating relations are only allowed *within* the units of the panel. The cointegration rank is estimated by sequentially testing

$$H(r) : rank(\Pi) \leq Nr \tag{5}$$

against the alternative

$$H(p) : rank(\Pi) \leq Np \tag{6}$$

as in Johansen (1995). A more detailed explanation of the test is provided in the appendix.

A limitation in the panel cointegration test by Larsson and Lyhagen (2007) is that the number of estimated parameters increases rapidly with the number of cross sections. This means that there needs to be enough time series observations compared to cross-sectional units, or the parameters of the model cannot be estimated. In our baseline dataset, there are nine countries and 37 time series observations per country, a relation which is far too small for the test. For this reason, the nine countries are divided in to three groups of countries according to their economic models. The groups are: Nordic, Central-European, and Anglo-Saxon countries.[8] Because countries in these groups tend to have similar economic and social structures, it is more likely that they also have homogenous cointegration relations.

Table 5 presents the results of the test for cointegration rank by Larsson and Lyhagen (2007) for the three groups of countries for gross savings and private consumption.[9] All variables are detrended and demeaned before testing. VAR lag lengths were determined using the Schwartz information criterion (SIC).

[8] Canada, United Kingdom and Unites States are included in the Anglo-Saxon group. France, Netherlands, and Switzerland are included in the Central-European group, and Finland, Norway, and Finland are included in the group of Nordic countries.

[9] All testing is done by Gauss. We are grateful to Johan Lyhagen for providing the Gauss code on his homepage.

Table 5: Panel trace cointegration test for savings, consumption and the top 1% income share for the 3 country groups

	Nordics	Central-Europe*	Anglo-Saxon**
depend. var.: gross savings			
top 1%, GDP & interest			
r=0	469.50	226.03	410.50
	(381.22)	(214.15)	(390.57)
r≤1	301.53	126.08	247.69
	(283.40)	(141.31)	(291.88)
r≤2	186.47	-	-
	(175.66)		
r≤3	113.70	-	-
	(83.32)		
depend. var.: private consumption			
top 1%, GDP & interest			
r=0	410.85	516.43	1137.26
	(381.27)	(470.88)	(992.44)
r≤1	281.61	194.42	586.90
	(267.77)	(214.56)	(523.78)
r≤2	172.84	-	300.25
	(166.60)		(336.24)
r≤3	58.99	-	-
	(89.81)		
countries	3	3	3
years	1960-03	1960-96	1960-00
observations	132	111	123

All series are detrended and demeaned before testing. * In the group of Central European countries, only GDP per capita and top 1% income share were included in the test, because there were too few time series observations per country to include a 4 variable. ** for Anglo-Saxon countries, GDP per capita in constant Euros was used instead of GDP per capita in purchasing power parities in the test with private consumption. All variables are tested in logarithms. Bartlett corrected critical values are presented in parentheses. Lag lengths were selected using Schwarz information criterion.

According to the results presented in the upper panel of table 5, there is one cointegration vector between gross savings, the GDP, the top 1% income share and the interest rate in Anglo-Saxon countries. In Central European countries, only gross savings, the top 1% income share, and the GDP per capita were included in the test, because there were not enough time series observations for including four variables. According to the results, the cointegration rank among these variables in Central-European countries is one.

According to the results of the Nordic countries presented in the upper panel of table 5, there are (at least) four stationary linear combinations with four explanatory variables. In the time series case this would imply that all variables are $I(0)$ trend-stationary. In panels, implications are not so straightforward. In the model (5), the block matrix elements of Π are given by $\Pi_{ij} = \Sigma_{k=1}^{N} \alpha_{ik}\beta'_{jk}$, which equal $\alpha_{ij}\beta'_{j}$ when $\beta_{ij} = 0$ for all $i \neq j$. However, if $\beta_{ij} \neq 0$ for $i \neq j$, then the block matrix elements of Π are given $\alpha_{ij}\beta'_{jk}$ and the rank of Π can be larger than the number of variables. That is, because the dimension of Π is $Np \times Np$, the number of cross-sectional cointegration relations may increase the rank of the matrix. If only gross savings, the interest rate and the top 1% are included in the test, two stationary linear combinations are found, i.e., a rank of two. If only gross savings and the GDP per capita are included, the test finds a rank of, at least, two. This implies that the GDP per capita series may be, in part at least, driven by a stationary linear combination of $I(1)$ non-stationary common factor(s). As Nordic countries have very similar social structures, and because they are small countries within close proximity to each other, it would be quite natural if a common stochastic trend would affect their GDP series. Thus, it seems that GDP per capita series in constant Euros are likely to be cross-sectionally cointegrated in Nordic countries.[10]

[10]Possible cross-unit cointegration may have affected panel unit root tests, but because cross-unit cointegration increases the likelihood of type I error, i.e., that null hypothesis of unit root is rejected wrongfully, cross-sectional cointegration has not biased the inference made from panel unit root tests (Banerjee *et al.*

According to the results presented in the lower panel of table 5, private consumption, the GDP per capita, the top 1% income share, and the interest rate would have a cointegration rank of two in the Anglo-Saxon countries and or cointegration rank of three in the Nordic countries. In Central European countries only private consumption, the top 1% income share and the GDP per capita were included in the testing, because there were not enough time series observations for including four variables. In Central-European countries the cointegration rank among these three variables is found to be one.

The results of Nordic countries implying the presence of a common stochastic trend driving the real GDP processes, unfortunately, raises an another dilemma. If income is affected by a common stochastic trend, it is likely to affect consumption as well, especially if the two variables are cointegrated (see previous section). To test this assumption, in table 6 we present the results of Larsson and Lyhagen (2007) panel cointegration test using just two variables, the income share of top 1%, and gross savings and private consumption each in turn.

According to the results presented in table 6, there are, at least, two stationary cointegration relations in the group of Nordic countries in both tests. As explained above, this is likely to indicate that gross savings and private consumption levels are cross-sectionally cointegrated across the Nordic countries.[11] In addition, the results also imply that the top 1% income share and gross savings would be difference stationary variables in the Central European and Anglo-Saxon countries. That is, both would be $I(1)$ variables that are *not* cointegrated with each other. Private consumption and the top 1% income share, on the other hand, are found to be cointegrated of order one in the Central European and Anglo-Saxon countries.

(2005)).

[11] In principle, top 1% income shares could also be cointegrated across cross-sections. However, it is more likely that the likely cross-sectional cointegration is related to consumption and savings through incomes that are driven by common stochastic trends.

Table 6: Panel trace cointegration test for savings, consumption and the top 1% income share for the 3 country groups

	Nordics	Central-Europe*	Anglo-Saxon**
depend. var.: gross savings			
top 1%			
r=0	211.12	292.15	72.32
	(197.64)	(307.22)	(92.86)
r≤ 1	57.52	-	-
	(50.71)		
depend. var.: private consumption			
top 1%			
r=0	295.33	294.36	194.75
	(167.08)	(270.17)	(171.47)
r≤1	68.33	72.97	38.86
	(51.49)	(83.63)	(59.64)
countries	3	3	3
years	1960-03	1960-96	1960-00
observations	132	111	123

All series are detrended and demeaned before testing. * In the group of Central European countries, only GDP per capita and top 1% income share were included in the test, because there were too few time series observations per country to include a 4 variable. ** for Anglo-Saxon countries, GDP per capita in constant Euros was used instead of GDP per capita in purchasing power parities in the test with private consumption, because... All variables are tested in logarithms. Bartlett corrected critical values are presented in parentheses. Lag lengths were selected using Schwarz information criterion.

As such the results of table 6 contradict the results presented in table 4, where gross savings and the top 1% income share were found to be cointegrated. Thus, the results of the cointegration tests for gross savings are somewhat inconclusive. It seems clear, however, that the top 1% income share and private consumption are cointegrated in all countries in the dataset.

The results of the panel cointegration rank tests imply that there might be long-run dependency relations in income and consumption series between the Nordic countries, which, naturally, could not be eliminated by allowing for short-run dependencies and/or cross-sectional correlation. Fortunately, Wagner and Hlouskova (2010) have found that, if there is only a cross-sectionally identical unit-specific cointegrating relationship(s) between cross-sections, it creates only a small bias in the results of used cointegration estimators.

5 Estimation

According to the results presented in the previous section, the cointegration rank between tested variables varies between the groups of countries. Because of this, two different estimation methods are applied to estimate the cointegration coefficient of the income share of the top 1%. Panel dynamic seemingly unrelated regressors estimator of Mark *et al.* (2005) is used when there seems to be only one cointegrating vector between the variables, and a two-step maximum-likelihood estimator of Breitung (2005) is used when there seem to be two or more cointegrating relations between the variables. The reason for using two estimators is that Wagner and Hlouskova (2010) have found that single-equation estimators, like panel DSUR, perform better, when cross-sections are cross-sectionally correlated and/or cointegrated.[12] Both estimators control

[12]Results of Wagner and Hlouskova (2010) imply that panel DOLS would perform better than panel VAR by Breitung (2005) in cross-sectionally cointegrated panels. Panel DSUR was not included in testing. However, as panel DSUR is more efficient than panel DOLS when cross-sections are correlated, it is also likely to be more efficient than panel DOLS when cross-sections are cointegrated.

for possible endogeneity of regressors. A more detailed explanation of the used estimators can be found in the appendix.

The estimated model is:

$$log(Y_{it}) = \alpha_i + \gamma_1' log(GDP_{it}) + \gamma_2' log(top1_{it}) + +\gamma_3'(interest_{it}) + \lambda_i t + u_{it}, \tag{7}$$

where where α_i's are individual constant, $\lambda_i t$'s are individual trends, and u_{it} is a white noise error vector with $E(u_{it}) = 0$. Table 7 presents the results of estimation of equation (7) using panel DSUR and panel VAR estimators.[13] In the last estimation dependency rate is also included in the estimation of equation (7).

According to the results presented in table 7, the control variables have a clear effect on the results of the estimation. In all groups, the initial estimate of the cointegrating coefficient of inequality measured with the top 1% income share is negative and statistically significant at the 0,1% level, but it changes to positive, when the GDP per capita is added to the estimation.[14] When the interest rate is added to the estimation, the estimate of inequality is statistically significant only in Anglo-Saxon and Nordic countries. However, as the panel cointegration test by Larsson and Lyhagen (2007) found that inequality and gross savings were not cointegrated in Anglo-Saxon countries, their estimation result may be spurious. So, accordingly, only the results of the Nordic countries are realiable. Last estimation adds also the dependency rate, which is commonly used to control for changes in saving patterns across the life cycle. In this last case, estimation results should be interpreted carefully, because adding dependency rate creates a possible bias in estimates as it may be a trend-stationary variable (see section 3.2).

Results presented in tables 7 and 6 imply that changes in income inequality do only have statistically significant effect on gross savings in group of Nordic countries. In

[13] Estimation was conducted with Gauss. Author is grateful to Donggyu Sul and Joerg Breitung for providing the program code on their homepages.

[14] If first equation is estimated with panel VAR, results are similar to those presented in table 7.

Table 7: Estimates of the long-run elasticity of gross savings with respect to top 1% income share in 3 groups of countries

Dependent variable: log(gross savings)

	Nordics	Central-Europe	Anglo-Saxon
Panel DSUR (leads & lags=2)			
log(top 1%)	-0.0846**	-0.127***	-0.4462
	(0.0288)	(0.0405)	(0.3706)
Panel VAR (lags=2;1;1)			
log(top 1%)	0.0799***	0.3794	0.2499
	(0.0251)	(0.2751)	(0.1769)
log(GDP)	0.0118***	0.0946***	0.0903***
	(0.0015)	(0.0100)	(0.0108)
Panel VAR (lags=2;1;2)			
log(top 1%)	0.5654*	0.1866	0.5385*
	(0.2512)	(0.2583)	(0.1990)
log(GDP)	0.1374***	0.0894***	0.0983***
	(0.0169)	(0.0097)	(0.0105)
log(interest)	-0.3655**	-0.0213	0.0990
	(0.1237)	(0.0477)	(0.0803)
Panel VAR (lags=4;3;3)			
log(top 1%)	0.3223	0.3845	0.4068**
	(0.2292)	(0.2162)	(0.1441)
log(GDP)	0.1175***	0.1036***	0.0819***
	(0.0017)	(0.0090)	(0.0072)
log(interest)	-0.396***	0.0947*	-0.0007
	(0.0754)	(0.0400)	(0.0588)
log(dependency)	-0.1006	0.1598	-0.2801***
	(0.1500)	(0.0865)	(0.0787)
countries	3	3	3
years	1960-03	1960-96	1960-00
observations	132	111	123

* = p<.05, ** = p<.01, *** = p<.001. Standard errors of the parameter estimates are presented in parentheses. Standard errors are estimated using Andrews and Monahan's Pre-whitening method. Inclusion of individual constants means that all estimations are made with fixed effects. Lags gives the lag order of the VAR model. Leads & lags=1 means that first lags and leads of first differences of explanatory variables are used as instruments. Leads & lags=2 means that first and second leads and lags of first differences are used as instruments, etc.

Central-European countries, income inequality had no statistically significant effect on savings and results of Anglo-Saxon countries may be a result of spurious regressions. The reason why gross savings and inequality would be cointegrated in the Nordic countries, but not in the Anglo-Saxon countries, is likely to be related to the different social structures between these groups of countries. In Nordic countries, changes in government consumption affect income distribution through extensive social security. This effect is likely to be considerable smaller in Anglo-Saxon countries, where social security is usually not so extensive as in Nordic countries. However, this does not explain why gross savings and the top 1% income share were not cointegrated in Central-European countries, who do have similar structures of social security. This result makes it likely that gross savings includes elements that are not related to possible changes in income inequality, e.g., goverment savings. Saving decisions of governments are likely to be driven mostly by other factors, like the business cycle, than changes in income inequality. This way income inequality could have only a marginal effect on gross savings.

Theories describing the relation between inequality and savings usually do concentrate on household savings behavior, which makes private savings or consumption a more valid measure to assess the effect of inequality on consumption or savings. Table 8 presents the results of panel DSUR and panel VAR estimations where the dependent variable is private consumption.[15]

According to the results presented in table 8, changes in income inequality would have a different impact on private consumption in different groups of countries. Once again, results including dependency rate needs to be taken only as suggestive, as it is possible that dependency rate is a stationary variable and including it to the estimation may bias the results. Thus, we rely more on the results presented in the 3rd estimation, which includes the top 1% income share, the GDP per capita, and the interest rate.

[15] Estimation was conducted with Gauss. Author is grateful to Donggyu Sul and Joerg Breitung for providing the program code on their homepages.

Table 8: Estimates of the long-run elasticity of private consumption with respect to top 1% income share in 3 groups of countries

Dependent variable: log(private consumption)

	Nordics	Central-Europe	Anglo-Saxon
Panel DSUR (leads & lags=2)			
log(top 1%)	-0.1051***	-0.1480***	-0.0704*
	(0.0119)	(0.0168)	(0.0304)
Panel VAR (lags=2;1;2)			
log(top 1%)	-0.1409**	-0.2101***	0.1130***
	(0.0490)	(0.0530)	(0.0290)
log(GDP)	0.0905***	0.1002***	0.0982***
	(0.0039)	(0.0026)	(0.0021)
Panel VAR (lags=1;1;2)			
log(top 1%)	-0.1349*	-0.1748***	0.0826***
	(0.0512)	(0.0451)	(0.0262)
log(GDP)	0.0919***	0.0996***	0.1007***
	(0.0042)	(0.0024)	(0.0021)
log(interest)	-0.0132	0.0203	-0.0383***
	(0.0262)	(0.0120)	(0.0108)
Panel VAR (lags=4;2;3)			
log(top 1%)	-0.1765***	-0.0731	0.0957***
	(0.0458)	(0.0387)	(0.0185)
log(GDP)	0.0878***	0.1005***	0.1005***
	(0.0038)	(0.0030)	(0.0022)
log(interest)	0.0082	-0.0057	-0.0297***
	(0.0230)	(0.0086)	(0.0069)
log(dependency)	0.0174	-0.0012	-0.0252
	(0.0305)	(0.0220)	(0.0150)
Panel VAR (lags=-;-;1)			
log(top 1%)	-	-	0.0852***
			(0.0202)
log(GDP)	-	-	0.0963***
			(0.0022)
log(interest)	-	-	-0.0350***
			(0.0084)
log(credit)	-	-	0.0026*
			(0.0010)
countries	3	3	3
years	1960-03	1960-96	1960-00
observations	132	111	123

* = p<.05, ** = p<.01, *** = p<.001. Standard errors of the parameter estimates are presented in parentheses. Standard errors are estimated using Andrews and Monahan's Pre-whitening method. Lags gives the lag order of the VAR model. Individual constants and trends are included in the regressions. Leads & lags=1 means that first lags and leads of first differences of explanatory variables are used as instruments. Leads & lags=2 means that first and second leads and lags of first differences are used as instruments, etc.

According to the results of this estimation, the GDP per capita has a expected positive sign in all groups of countries, but the interest rate has a statistically significant negative effect only in the group of Anglo-Saxon countries. The "blurry" estimate of interest rates in Central-European countries is not a surprise as these countries had a differing indicators of interest rates. In Anglo-Saxon and Nordic countries, individual countries had the same indicator of interest rates within groups, but the indicator differed between groups. In Anglo-Saxon countries, treasury bill rate was used, whereas in Nordic countries, central bank rate was used. Although, it is surprising that results indicate that interest rates had no statistically significant effect on the level of private consumption in Nordic countries, result may be explained by different consumption profiles. Bacchetta and Gerlach (1997) found that in United States and Canada the changes in credit conditions had a larger impact on consumption than in France or UK. Humphrey (2004) also shows that credit cards are used more often as a means of payment in Canada and US than in Europe. Thus, it is likely that changes in the interest rates have a greater effect in private consumption in Anglo-Saxon than in European countries.

According to the results of estimation including the top 1% income share, the GDP per capita, and the interest rate, the long-run elasticity of private consumption with respect to inequality would be negative in Nordic and Central-European countries, but in Anglo-Saxon countries, the effect would be positive. To check that this 'anomaly' is not caused by expansion of credit in the wake of rising inequality, domestic credit claims on the private sector is added as an explanatory variable to the estimation of Anglo-Saxon countries.[16] The variable is obtained from IMF database. Unfortunately, there were no data for Central-European and Nordic countries for the whole time period, and so domestic credit claims could not be included in their estimations. In the Anglo-Saxon case, logarithmic domestic credit claims is found to be $I(1)$ according to

[16]The variable is expressed in purchasing power parities.

all panel unit root tests used in section 3. Adding a measure of credit does not change main results for Anglo-Saxon countries and it has expected statistically significant positive effect on private consumption.[17]

Thus, results presented in table 8 imply that saving functions of individuals would differ between Anglo-Saxon and European countries. In Anglo-Saxon countries, inequality of income increases aggregate private consumption whereas in Central-European and Nordic countries inequality was found to decrease private consumption. This means that in European countries individual saving function would be convex, whereas in Anglo-Saxon countries it would be concave. This result links this study to the long line of research on the variations in the propensity to save across income classes starting, at least, from Fisher (1930). Result obtained here also contradict some quite recent microeconomic evidence from US stating that propensity to save raises with income (Dynan *et al.* 2004). However, as was presented in the introduction, results of aggregate studies have been mixed. Results presented here imply that some of this observed controversy in previous macro studies might be explained by country differences. Despite of this, it is unclear of why these countries would differ in this respect and there is a possibility that the results of this study are driven by some features of the data that could not be identified and controlled with the used estimation methods. If the model (7) is estimated without the GDP per capita series, the cointegrating coefficient of inequality in Anglo-Saxon countries changes from positive to negative (-0.044) and remains statistically significant. The GDP per capita series only has this effect in Anglo-Saxon countries, which implies that GDP per capita series may be the source of this possible bias. One possible source of such bias could be the existence of non-identical cross-sectional cointegration relations in the GDP per capita series between Anglo-Saxon countries. In this case, estimators used to estimate equation (7) are likely

[17] Results do not change if dependency rate is added to estimation.

to be adversely affected (Wagner and Hlouskova 2010).[18] Unfortunately, we do not know any way of testing this assumption within the limited time series extent of our data.[19] Therefore, more empirical research is needed to find are the results for Anglo-Saxon countries presented here driven by such cross-sectional dependency relations.

6 Conclusions

According to the results, income inequality, measured with the top 1% income share, aggregate savings and private consumption are driven by stochastic processes. The top 1% income share and private consumption were also found to be cointegrated, but top 1% income share and gross savings was found to be cointegrated only in Nordic countries. The long-run elasticity of private consumption with respect to inequality was found to be negative in Central-European and Nordic countries, but positive in Anglo-Saxon countries. This result implies that, in Anglo-Saxon countries, the marginal propensity to save would decrease with income, while in European countries it would increase. However, results of Anglo-Saxon countries crucially depend on the inclusion of the GDP per capita series. If the GDP per capita was not included in the estimation, the cointegrating coefficient of the top 1% income share was negative and statistically significant. This implies that the inclusion of the GDP per capita series may introduce such bias in the estimation results of Anglo-Saxon countries which cannot be controlled with current panel estimation methods.

Individual cointegration relations in the GDP per capita series that are not identical across the Anglo-Saxon countries could be one source of such bias. These kind of dependency relations would cause estimators to be biased in ways that are not yet exactly known. The existence of such non-identical cointegration relations could not be tested,

[18]We, or Wagner and Hlouskova (2010), do not know any estimation method that would correct for this kind of cross-sectional cointegration.

[19]There is a way to test for cointegration relations between countries using a test for cointegrated systems by Gonzalo and Granger (1995). However, the test requires that we have a lot more time series observations per country than we have in our dataset.

because testing would have required considerably more time series observations that were available in our dataset. So, results for Anglo-Saxon countries remain ambiguous and this ambiguity needs to be addressed in future studies.

In Nordic and Central-European countries, individual cointegrating relations may not have been identical within groups of countries, which could have biased their estimation results. However, if this assumption is violated, it would imply that the underlying economic theory is not applicable across countries with similar economic structures questioning the validity of the theory. Estimation results were also unchanged when only the top 1% income share was used as an explanatory variable. In this case there can be only one cointegration vector between variables, which implies that if the number of cointegration vectors has differed within the groups of countries in estimations with several explanatory variables, this has not changed basic results.

So, although results of Anglo-Saxon countries are somewhat inconclusive, results for Nordic and Central-European countries clearly indicate that income inequality leads to greater level of private savings. This result is well in-line with the theoretical and micro-econometric evidence. It also implies that the controversy surrounding the result of previous aggregate studies has been likely to result from a miss-specification of the estimated models by assuming a stationary income inequality.

APPENDIX

Appendix A: Cointegration test by Banerjee and Carrion-I-Silvestre (2006)

Panel cointegration test developed by Banerjee and Carrion-i-Silvestre (2006) is based on the normalized bias and the pseudo t-ratio test statistics by Pedroni (2004). The data generating process behind Pedroni's test statistics is given by:

$$y_{it} = f_i(t) + x'_{it} + e_{it},$$

$$\triangle x_{it} = v_{it}, \tag{8}$$

$$e_{it} = \rho_i e_{i,t-1} + \epsilon_{it}\zeta_{it} = (\epsilon_{it}, v_{it})',$$

where $f_i(t)$ includes member specific fixed effects and deterministic trends.

The data generating process is described as a partitioned vector $z'_{it} \equiv (y_{it}, x_{it})$ where the true process is generated as $z_{it} = z_{i,t-1} + \zeta_{it}$, $\zeta'_{it} = (\zeta^y_{it}\zeta^X_{it})$ (Pedroni 2004). $\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]}\zeta_{it}$ is assumed to converge to a vector Brownian motion with asymptotic covariance of Ω_i as $T \longrightarrow \infty$. The individual process is assumed to be *i.i.d.* so that $E[\zeta_{it}\zeta'_{js}] = 0$ $\forall s, t, i \neq j$.

Let $\hat{e}_{it}$ denote the estimated residuals of obtained from (8) and $\hat{\Omega}_i$ the consistent estimator of Ω_i. The two test statistics can now be defined as :

$$\tilde{Z}_{\hat{\rho}_{NT}-1} \equiv \sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{e}^2_{i,t-1}\right)^{-1}\sum_{t=1}^{T}(\hat{e}_{i,t-1}\Delta\hat{e}_{it} - \hat{\lambda}_i),$$

$$\tilde{Z}^*_{\hat{t}_{NT}} \equiv \sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{s}^{*2}_i\hat{e}^{*2}_{i,t-1}\right)^{-1/2}\sum_{t=1}^{T}(\hat{e}^*_{i,t-1}\Delta\hat{e}^*_{it}),$$

where $\hat{\lambda}_i = 1/T\sum_{s=1}^{k_i}(1 - s/(k_i+1))\sum_{t=s+1}^{T}\hat{\mu}_{it}\hat{\mu}_{i,t-s}$, $\tilde{\sigma}^2_{NT} \equiv 1/N\sum_{i=1}^{N}\hat{L}^{-2}_{11i}\hat{\sigma}^2_i$, $\hat{s}^{*2}_i \equiv 1/t\sum_{t=1}^{T}\hat{\mu}^{*2}_{it}$, $\tilde{s}^{*2}_{NT} \equiv 1/N\sum_{i=1}^{N}\hat{s}^{*2}_i$, $\hat{L}^2_{11i} = 1/T\sum_{t=1}^{T}\hat{\vartheta}^2_{it} + 2/T\sum_{s=1}^{k_i}(1 - s/(k-i+1))\sum_{t=s+1}^{T}\hat{\vartheta}_i, \hat{\vartheta}_{i,t-s}$. The residuals $\hat{\mu}_{it}$,$\hat{\mu}^*_{it}$ and $\hat{\vartheta}_{it}$ are attained from regressions: $\hat{e}_{it} = \hat{\gamma}\hat{e}_{i,t-1} + \hat{\mu}_{it}$, $\hat{e}_{it} = \hat{\gamma}_i\hat{e}_{i,t-1} + \sum_{k=1}^{K-i}\hat{\gamma}_{ik}\Delta\hat{e}_{i,t-k} + \hat{\mu}^*_{it}$, $\Delta y_{it} = \sum_{m=1}^{M}\hat{b}_{mi}\Delta x_{mi,t} = \hat{\vartheta}_{it}$. (Pedroni 1999, 2004)

The statistics pool the between dimension of the panel and they are constructed by computing the ratio of the corresponding conventional time series statistics and then by computing the standardized sum of the N time series of the panel. Pedroni (1999,

2004) shows that under the null of no cointegration the asymptotic distributions of the two statistics presented above converge to normal distributions with zero mean and variance of one as N and T sequentially converge to infinity.

Banerjee and Carrion-I-Silvestre (2006) extend the model by Pedroni (2004) to include common factors:

$$\begin{aligned} y_{i,t} &= f_i(t) + x'_{i,t} + u_{i,t}, \\ \triangle x_{i,t} &= v_{i,t}, \\ f_i(t) &= \mu_i + \beta_i t + \theta DU_{i,t} + \gamma_i DT^*_{i,t}, \\ u_{it} &= F'_t \pi_i + e_{it} \end{aligned} \tag{9}$$

where $e_{i,t} = \rho_i e_{i,t} + \epsilon_{i,t}$,

$$DU_{i,t} = \begin{cases} 0 & t \leq T_{bi} \\ 1 & t > t_{bi} \end{cases}, \tag{10}$$

$$DT^*_{i,t} = \begin{cases} 0 & t \leq T_{bi} \\ (t - t_{bi}) & t > t_{bi} \end{cases}, \tag{11}$$

where $T_{bi} = \lambda_i T$, $\lambda_i \in \Lambda$, denotes the time of the break for the i-th unit in a closed subset of (0,1), and F'_t:s are the common factors which are used to account for the possible cross-sectional dependence. The cointegrating vector is specified as a function of time:

$$\delta_{i,t} = \begin{cases} \delta_{i,1} & t \leq T_{bi} \\ \delta_{i,2} & t > t_{bi} \end{cases}. \tag{12}$$

Banerjee & Carrion-i-Silvestre's test computes a $Z^e_{\bar{t}_{NT}}(\lambda) = N^{-1} \sum_{i=1}^N t_{\hat{\rho}_i}(\lambda)$ statistic for each break point using the idiosyncratic disturbance terms (e_{it}). The break point is estimated as the argument that minimizes the sequence of standardized statistics. Thus, the estimated break date is given by

$$\hat{T}_b = arg \min_{\lambda \in \Lambda} \left(\frac{N^{-1/2} Z^e_{\bar{t}_{NT}}(\lambda) - \Theta^e_2(\lambda)\sqrt{N}}{\sqrt{\psi^e_2(\lambda)}} \right).$$

APPENDIX B: Panel trace cointegration test statistic by Larsson and Lyhagen (2007)

The trace cointegration test by Larsson and Lyhagen (2007) is based on the following model:

$$\triangle Y_t = \mu + \Pi Y_{t-1} + \Sigma_{k=1}^{m-1} \Delta Y_{t-k} + \epsilon_t, \tag{13}$$

where $\mu = (\mu_1', \mu_2', ..., \mu_N')'$, $\epsilon_t = (\epsilon_{1t}', \epsilon_{2t}', ..., \epsilon_{nt}')$, Y_{t-1} and ΔY_{t-k} are of order $Np \times 1$, Π and Γ_k are $Np \times Np$, and ϵ_t is assumed to be multivariate normally distributed with mean zero and covariance matrix Ω_{ij}.

It is assumed that matrix Π has a reduced rank of Nr, $0 \leq r \leq p$, which is specified as $\Pi = \alpha_{ik}\beta_{kj}'$ (Larsson and Lyhagen 2007). Matrices α and β are both order of $Np \times Nr$ and the former contains the short-run coefficient and the latter the long-run coefficient. In β, $\beta_{ii} \equiv \beta_i$ for each rank of r. Because of the restriction, $\beta_{kj} = 0$ $\forall i \neq j$, the block matrix elements of Π are given by $\Sigma_{k=1}^{N} \alpha_{ik}\beta_{kj}' = \alpha_{ij}\beta_j'$.

The cointegration rank is estimated by sequentially testing

$$H(r) : rank(\Pi) \leq Nr \tag{14}$$

against the alternative

$$H(p) : rank(\Pi) \leq Np, \tag{15}$$

which is the same method as in Johansen (1995).

Define Q_T as the maximum likelihood ratio test statistic for the test of $H(r)$ against $H(p)$, and assume that the matrix $\alpha' \perp \Gamma\beta \perp$ has a full rank and that the roots of the characteristic polynomial

$$A(z) = (1-z)I_{Np} - \alpha\beta' z - \Sigma_{i=1}^{m-1} \Gamma_k (1-z) z i \tag{16}$$

lie outside the complex unit circle. Now, if $r > 0$,

$$\text{-2 } \log Q_t \xrightarrow{w} U + V, \tag{17}$$

as $T \longrightarrow \infty$, where V is χ^2 with $N(N-1)r(p-r)$ degrees of freedom independent of U, and

$$U = tr\left\{\int dBF'\left(\int FF'\right)^{-1}\int F\mathrm{d}B'\right\}. \tag{18}$$

Larsson and Lyhagen (2007) show that the limit distribution of the test statistic (17) equals the convolution of Dickey-Fuller distribution (B) and an independent χ^2 variate (F). The distribution can be simulated by approximating the Wiener process of B by a random walk.

APPENDIX C: Panel DSUR and panel VAR estimators

The data generation process in Mark *et al.* (2005) DSUR estimator is of the form

$$y_{it} = \alpha_i + \lambda_i t + \theta_t + \beta' x_{it} + u_{it}, \tag{19}$$

$$\triangle x_{it} = e_{it} \tag{20}$$

where there are n cointegrating regression each with T observations, $(1-\beta')$ is the cointegration vector between y_{it} and x_{it}, and x_{it} and e_{it} are $k \times 1$ dimensional vectors. Panel DSUR eliminates the possible endogeneity between explanatory variables and the dependent variable by assuming that u_{it} is correlated at most with p_i leads and lags of $\triangle x_{it}$ (Mark *et al.* 2005). The possible endogeneity can be controlled by projecting u_{it} onto these p_i leads and lags:

$$u_{it} = \sum_{s=-p_i}^{p_i} \delta'_{i,s} \triangle x_{i,t-s} + u_{it}* = \delta'_i z_{it} + u^*_{it}. \tag{21}$$

The projection error u^*_{it} is orthogonal to all leads and lags of $\triangle x_{it}$ and the estimated equation becomes:

$$y_{it} = \alpha_i + \lambda_{it} + \theta_t + \beta' x_{it} + \delta_i z_{it} + u^*_{it}, \tag{22}$$

where $\delta'_i z_{it}$ is a vector of projection dimensions. Panel DSUR estimates a long-run covariance matrix that is used in estimation of equation (19). This makes panel DSUR more efficient when cross-sections are dependent. The efficiency of panel DSUR actually improves as the correlation between cross-sections increases. Asymptotics properties of the estimator are based on $T \longrightarrow \infty$ with N fixed.

In the estimator by Breitung (2005) a panel VAR(p) model is presented in VECM form as

$$\triangle y_{it} = \Psi_i d_t + \alpha_i \beta'_{y,t-1} + \Sigma_{j=1}^{p-1} \Gamma_{ij} \triangle y_{i,t-j} + \epsilon_{it}, \tag{23}$$

where d_t is a vector of deterministic variables and ψ_i a $k \times k$ matrix of unknown coefficients, Γ_{ij} is unrestricted matrix, and ϵ_{it} is a white noise error vector with $E(\epsilon_{it}) = 0$ and positive definite covariance matrix $\Sigma_i = E(\epsilon_{it}\epsilon'_{it})$.

First models are estimated separately across N cross-section units. Then cointegration vectors are normalized so that they do not depend on individual specific parameters. In the second stage, the system is transformed to a pooled regression of the form:

$$\hat{z}_{it} = \beta' y_{i,t-1} + \hat{v}_{it}, \tag{24}$$

where $\hat{z}_{it} = (\hat{\alpha}'_i \hat{\Sigma}_i^{-1} \hat{\alpha}_i)^{-1} \hat{\alpha}'_i \hat{\Sigma}_i^{-1} \triangle y_{it}$ and $\hat{v}_{it}$ is estimated from a transformed VECM model. It is assumed that statistical units included in the panel have the same cointegration rank. Cross-sectional correlation is accounted by using a estimated asymptotic covariance matrix. (Breitung 2005)

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